Fluid Motion

Bernoulli Equation

Assuming steady, incompressible flow and integrating Euler equation along a pathline, we have

Here we replace $dl = ds \quad and \quad a_l = a_t$, then Euler Equation becomes,

$$-\frac{d}{ds}(p+\gamma z) = \rho a_t$$

But

$$a_t = \left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

$$-\frac{d}{ds}(p+\gamma z) = \rho\left(V\frac{\partial V}{\partial s} + \frac{\partial V}{\partial t} \right)$$

For a steady flow, $\left(\frac{\partial V}{\partial t} \right) = 0$

$$-\frac{d}{ds}(p+\gamma z) = \rho\left(V\frac{\partial V}{\partial s} \right) = \rho\frac{d}{ds}\left(\frac{V^2}{2} \right)$$

Fluid Motion

$$-\frac{d}{ds}\left(p+\gamma z+\rho\frac{V^2}{2}\right)=0$$

$$\left(p+\gamma z+\rho\frac{V^2}{2}\right)=C$$

Equation (4.19) is called the Bernoulli's Equation which states that

The sum of piezometric pressure $(p+\gamma z)$ and the kinetic or dynamic pressure $(\rho\frac{V^2}{2})$

is equal constant for a steady, incompressible, inviscid fluid,

Another form of Bernoulli's Equation can be expressed as follows,

$$\left(\frac{p}{\gamma}+z+\frac{V^2}{2g}\right)=\left(h+\frac{V^2}{2g}\right)=C$$

Where

$$h = Piezometric\ head \quad and \quad \left(\frac{V^2}{2g}\right)=Dynamic\ head$$

Stagnation Tube

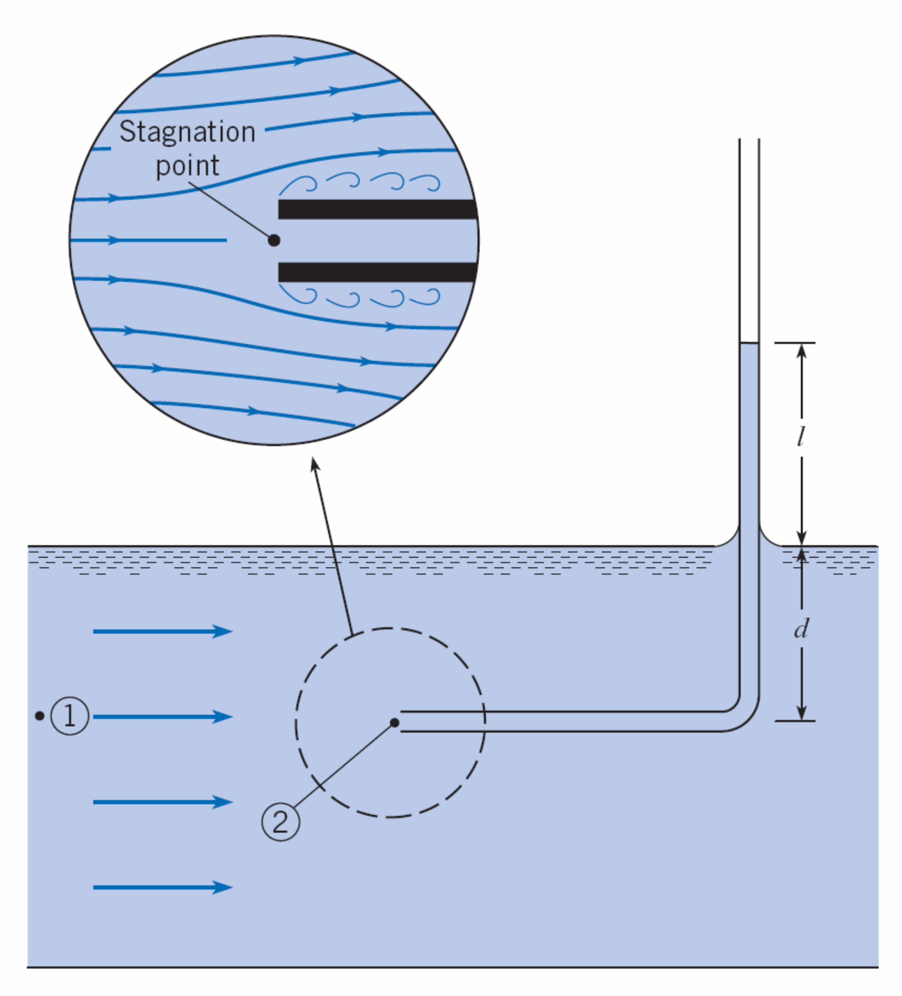
Applying Bernoulli's Eqn. between points (1) and (2), we have,

$$\left(p_1 + \gamma z_1 + \rho \frac{V_1^{\,2}}{2}\right) = \left(p_2 + \gamma z_2 + \rho \frac{V_2^{\,2}}{2}\right)$$

$$z_1 = z_2$$

$$\left(p_1 + \rho \frac{V_1^{\,2}}{2}\right) = \left(p_2 + \rho \frac{V_2^{\,2}}{2}\right)$$

Since $V_1 = 0$ (a stagnation point)



$$\left(\frac{V_1^{\,2}}{2}\right) = \frac{2}{\rho}(p_2 - p_1)$$

$$p_2 = \gamma(l + d) \quad and \quad p_1 = \gamma d$$

$$V_1 = \sqrt{2gl}$$

Pitot Tube

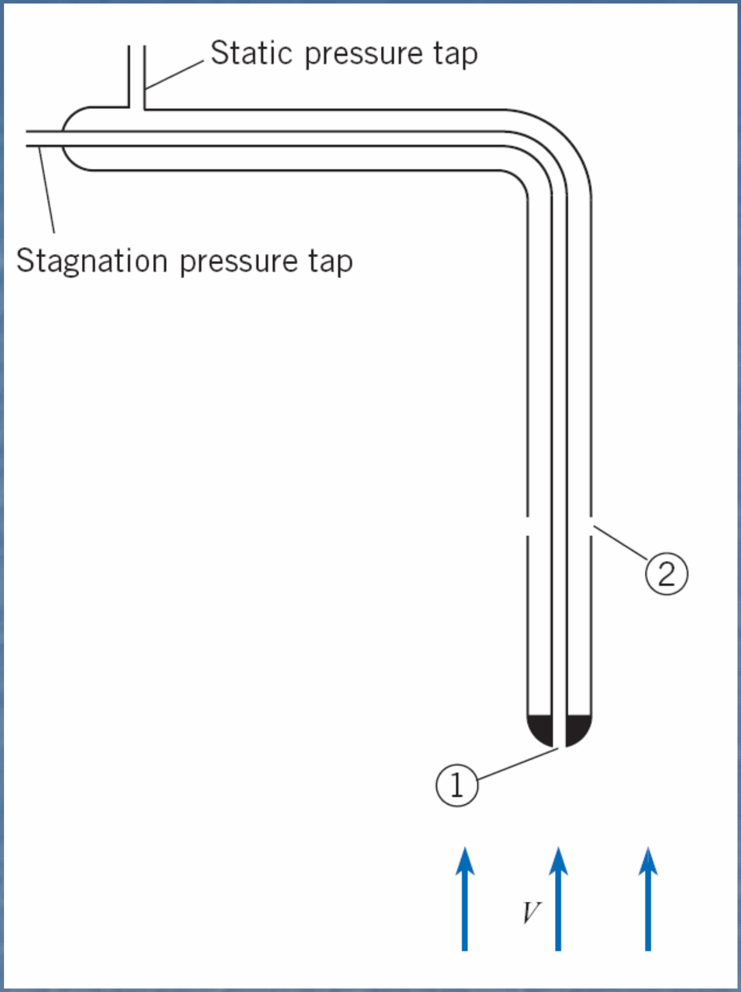
Applying Bernoulli's Eqn. between points (1) and (2), we have,

$$\left(p_1 + \gamma z_1 + \rho \frac{V_1^{\ 2}}{2}\right) = \left(p_2 + \gamma z_2 + \rho \frac{V_2^{\ 2}}{2}\right)$$

$$V_1 = 0$$

$$V_2 = \left[\frac{2}{\rho}\left(p_{z,1} - p_{z,2}\right)\right]^{1/2}$$

$$V_2 = V \quad \textit{Where } V \textit{ is the stream velocity}$$



Solved Example

Calculate $(V_{ker\,o})$ in the pipe

Applying Bernoulli's equation between points 1 and 2, we have

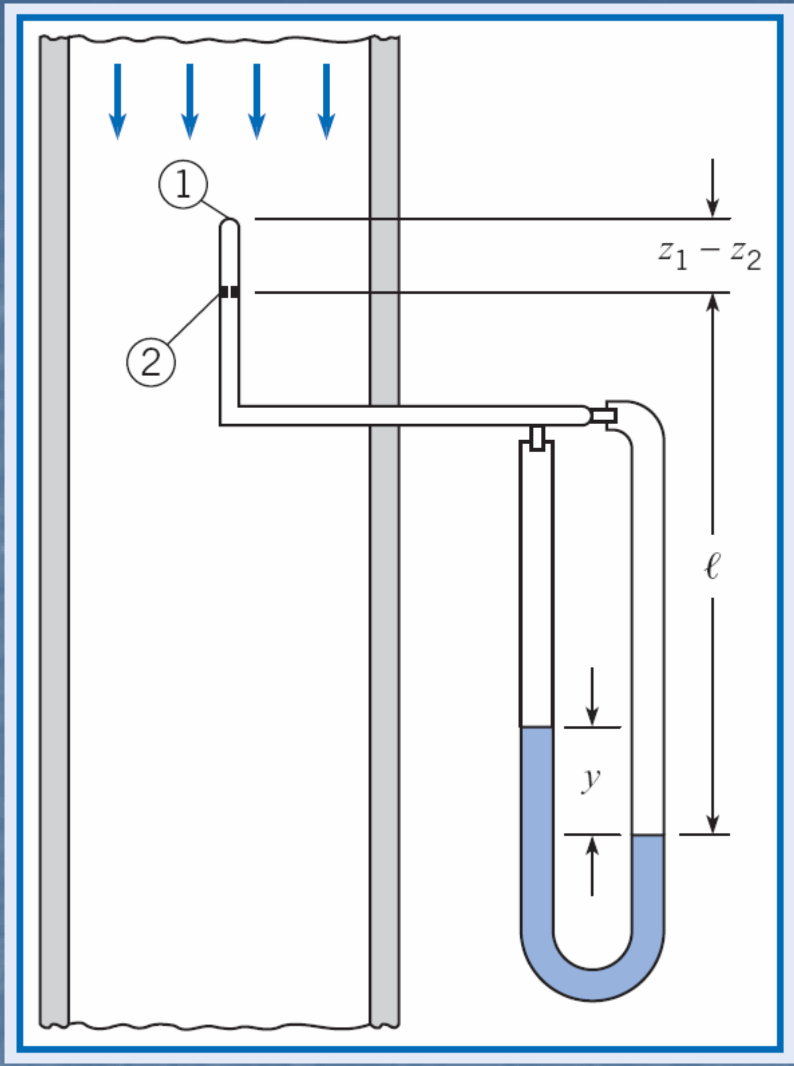
$$V_{ker\,osene} = \sqrt{\frac{2}{\rho_{ker\,osene}}(p_1 + \gamma_{ker\,o} z_1) - (p_2 + \gamma_{ker\,o} z_2)}$$

By applying the manometer equation between points 1& 2, we have,

$$p_2 = p_1 + \gamma_{ker\,o}(z_1 - z_2) + \gamma_{ker\,o} l - y\gamma_{Hg} - \gamma_{ker\,o}(l - y)$$

$$(p_1 + \gamma_{ker\,o} z_1) - (p_2 + \gamma_{ker\,o} z_1) = y(\gamma_{Hg} - \gamma_{ker\,o} y)$$

$$V_{ker\,osene} = \sqrt{\frac{2}{\rho_{ker\,osene}} y(\gamma_{Hg} - \gamma_{ker\,o})}$$



Fluid Motion

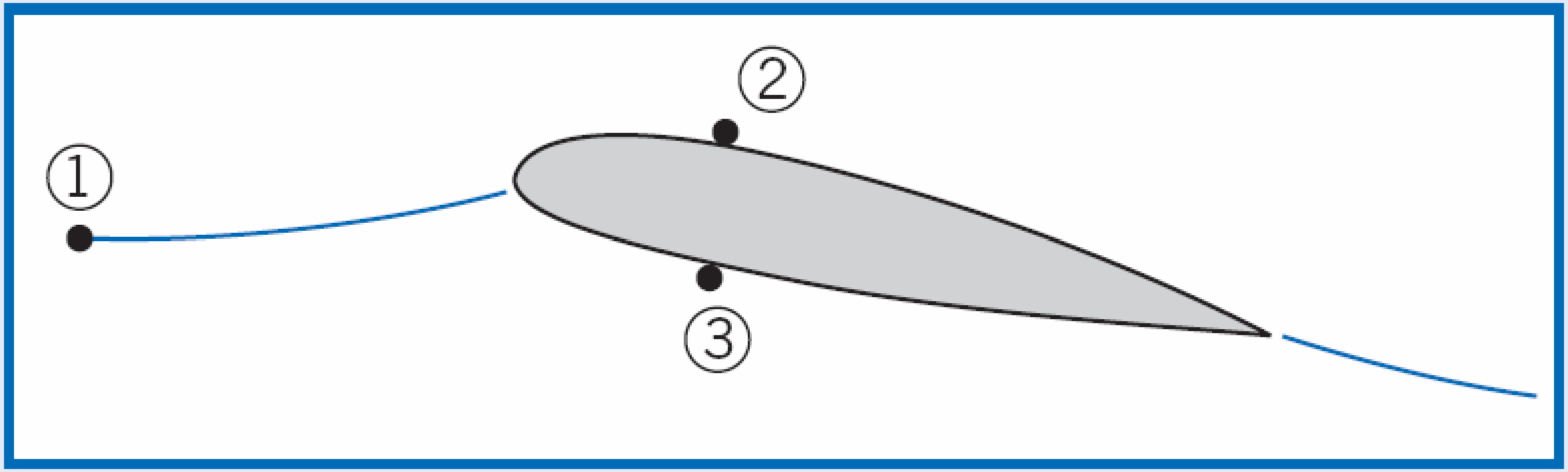
Pressure Coefficient

For Gases, Pressure coefficient is given as $C_p = \dfrac{p - p_0}{\frac{1}{2}\rho V_0^{\,2}}$

Where p_0 and V_0 are reference pressure and temperature.

For Liquids, Pressure coefficient is given as $C_p = \dfrac{h - h_0}{\frac{1}{2}\rho V_0^{\,2}}$

Where h is the piezometric head



Applying Bernoulli's equation between 1 & 2, we have $p_1+\rho\frac{V_1^{\,2}}{2}=p_2+\rho\frac{V_2^{\,2}}{2}$

Applying Bernoulli's equation between 1 & 3, we have $p_1+\rho\frac{V_1^{\,2}}{2}=p_3+\rho\frac{V_3^{\,2}}{2}$

$$p_2+\rho\frac{V_2^{\,2}}{2}=p_3+\rho\frac{V_3^{\,2}}{2}$$ $$p_3-p_2=\frac{\rho}{2}\left(V_2^{\,2}-V_3^{\,2}\right)$$

Applying Bernoulli's equation between 1 & 2, we have

$$p_1+\rho\frac{V_1^{\,2}}{2}=p_2+\rho\frac{V_2^{\,2}}{2}$$ $$p_2-p_1=\frac{\rho}{2}\left(V_1^{\,2}-V_1^{\,2}\right)$$

But $$C_{p2}=\frac{p_2-p_1}{\frac{1}{2}\rho V_1^{\,2}}$$

$$C_{p_2}=1-\left(\frac{V_2}{V_1}\right)^2$$ $$C_{p_3}=1-\left(\frac{V_3}{V_1}\right)^2$$

END OF LECTURE (5)